Student Name: ________________________

Math Class: ___________________________

James Ruse Agricultural High School

Year 12 Trial HSC Examination 2022

Mathematics Advanced

General Instructions

- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11-39, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I

- Attempt questions 1-10
- Answer on the multiple choice answer sheet provided
- Allow approximately 15mins for this section

Sections II, III and IV

- Attempt questions 11-39
- Answer on the space provided on the booklet
- Allow approximately 2hrs 45mins for these three sections.

Reading time: 10 minutes

Working time: 3 hours

Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

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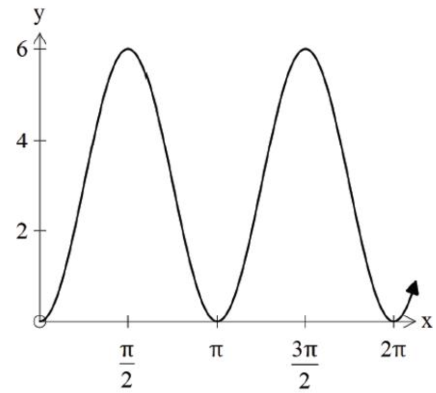
1. The equation of the graph below is given by $y = A\cos Bx + 3$.



Which of the following are the values of A and B?

A. $A = 3, B = 2$
B. $A = 6, B = 3$
C. $A = -3, B = 2$
D. $A = -6, B = \pi$

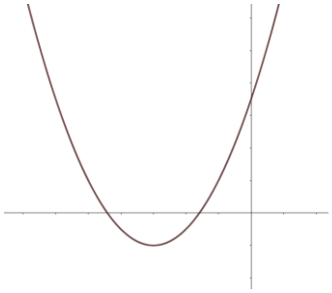
2. A data set of thirteen scores has a mean and median of 25. The scores 21, 16, 16, 26, 30, 30 are added to this data set.

What of the following MUST BE TRUE?

A. The mean increases
B. The mean decreases
C. The median increases
D. The median decreases

3. Which diagram below best shows the graph $y = 2 - (x+3)^2$?

A.



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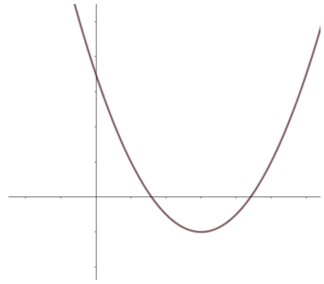
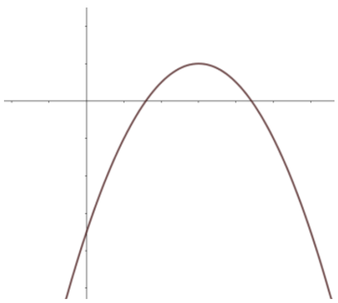
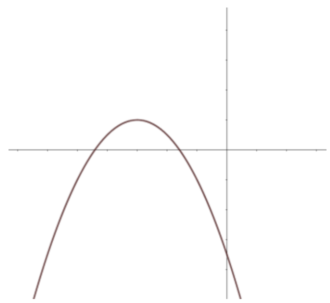


C.



D.



4. The following back-to-back stem and leaf plot displays the test results of a class of 24 students.

Boys					Girls				
			1	2	1	2	4		
			3	3	0	2	3	5	
	9	7	4	4	4	5	5	9	9
4	6	2	2	5	3				
				6	1	9			

What is the median and the mode for the class?

A. Median is 45 and Mode is 49

B. Median is 46 and Mode is 44

C. Median is 46 and Mode is 49

D. Median is 45 and Mode is 44

5. Which of the following is not equivalent to $\sqrt{(x-1)^2}$?

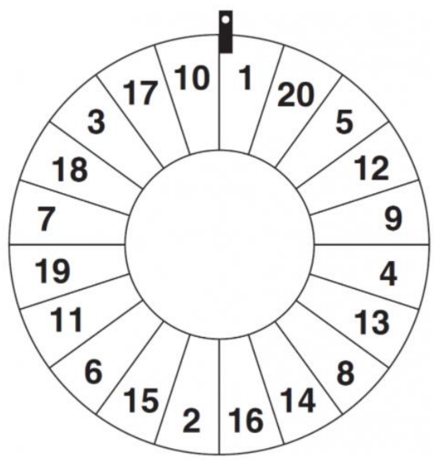
A. $|1-x|$

B. Distance from x to 1 on the number line?

C. $\begin{cases} x-1 & when\ x \geq 1 \\ 1-x & when\ x < 1 \end{cases}$

D. $x-1$

6. The wheel below displays numbers 1-20 exactly once for each number.



If the wheel is spun 160 times, how many times would you expect a number greater than 15 to be obtained?

A. 24

B. 32

C. 40

D. 48

7. What is $\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{2}+h\right)-\cos\left(\frac{3\pi}{2}\right)}{h}$?

A. -1

B. 0

C. 1

D. Undefined

8. The temperature of a room, in degrees Celsius, is modelled by $T(t)$ where t is the number of minutes after its thermostat is adjusted. What is the best interpretation of $T'(5) = 2$?

A. The temperature of the room is increasing at a rate of 2 degrees Celsius per minute, 5 minutes after the thermostat is adjusted.

B. The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Celsius per minute.

C. The temperature of the room increases by 2 degrees Celsius during the first 5 minutes after the thermostat is adjusted.

D. The temperature of the room is 2 degrees, 5 minutes after the thermostat is adjusted.

9. The graph of $y = \frac{3}{x+1}$ is translated 4 units right and dilated vertically by a factor of $\frac{1}{2}$. Which of the following gives the equation of the new function?

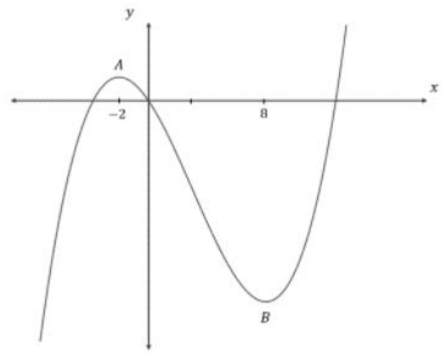
A. $\frac{y}{2} = \frac{3}{x-3}$

B. $2y = \frac{3}{x-4}$

C. $2y = \frac{3}{x-3}$`

D. $\frac{y}{2} = \frac{3}{x-4}$

10. The following diagram of $y = f(x)$, has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 8$.



What is the order of $f(-2)$, $f'(8)$, $f''(-2)$ in ascending order?

A. $f(-2), f'(8), f''(-2)$
B. $f''(-2), f'(8), f(-2)$
C. $f(-2), f''(-2), f'(8)$
D. $f'(8), f''(-2), f(-2)$

End Section I

Mathematics Advanced
Sections II, III, IV Answer Booklet

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section.

Instructions

- At the beginning of each section, write your Student Number at the top of the page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Additional writing spaced is provided at the back of the booklet. If you use this space, clearly indicate which question you are answering.

Section II (30 marks) Student Number:__________________

11. Find the largest domain for which $\sqrt{2x-3}$ is defined. **1**

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12. Factorise $2x^2 + 3x - 2xy - 3y$. **1**

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13. Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$.

a) Find $f(3)$. **1**

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b) Express $f(g(x))$ in the form $ax^4 + bx^2 + c$, where a, b and c are non-zero integers. **2**

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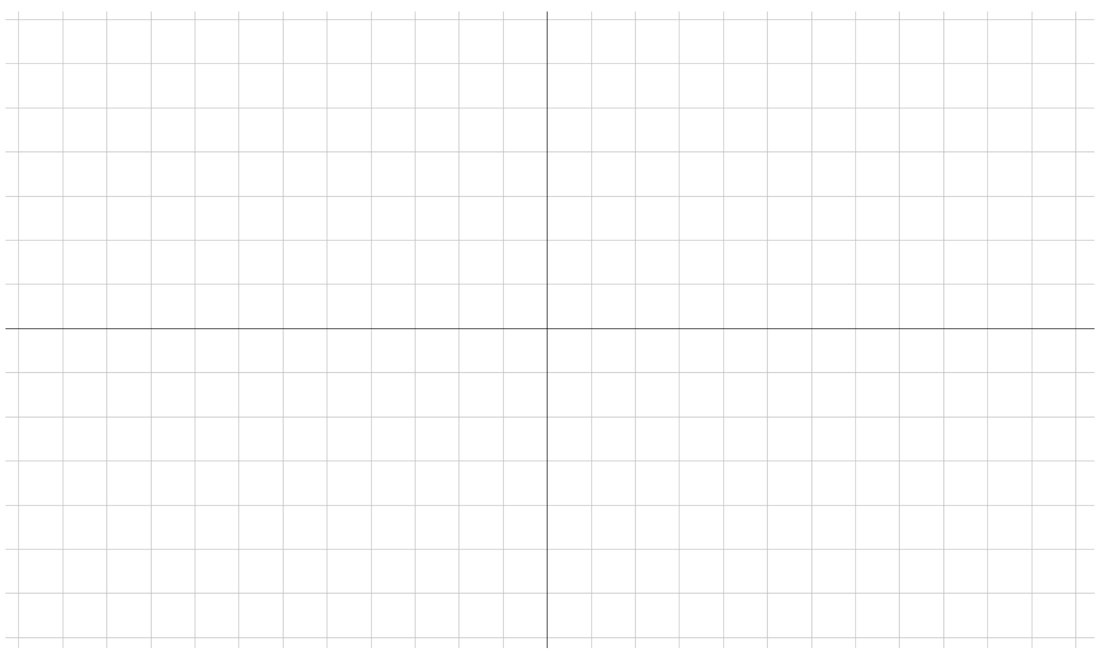
14. A circle is given by $x^2 + 2x + y^2 - 4y - 4 = 0$.

a) By completing the square, find the radius and centre of the circle. **2**

b) This circle is now reflected about the x-axis. Sketch the reflected circle. Do not solve for the coordinate intercepts. **2**



15. Rewrite $\sqrt{7} + \sqrt{8}$ in the form $\sqrt{a + b\sqrt{c}}$ using the smallest integer values possible for c. **3**

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16. An arithmetic sequence is given by $4, 13, 22, 31 \dots$.

a) Find the 25^{th} term of the sequence. **2**

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b) Find the sum to the 25^{th} term of the sequence. **1**

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17. Given that $4, x, y, \frac{32}{27}$ are four consecutive terms of a geometric series. Solve for x and y. **3**

18.

a) Show that the derivative of $\frac{x}{e^x}$ is $\frac{1-x}{e^x}$. **1**

b) Hence find $\int \frac{1-x+e^x}{2e^x}\,dx$. **2**

19. Find the derivative of $x^2 - 1$ using first principles. **3**

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20. The velocity of a particle travelling in a straight line is given by $v = 2t - 4$, where t is time in seconds and x its displacement from the origin in metres. The particle is initially at the origin.

a) Find the expression for the particle's displacement as a function of time. **1**

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Question 20 continues the next page

b) Find the total distance travelled for the first 5 seconds. **2**

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21. Solve for k such that $\int_2^k (-3x+1)\,dx = \int_k^6 (-3x+1)\,dx$. **3**

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End Section II

Extra Writing Space (Clearly indicate which Question you are doing).

Section III (30 marks) Student Number:__________________

22. The data from a weather balloon measuring the air temperature every kilometre as it rises through the atmosphere are shown below:

Altitude (km)	0	1	2	3	4	5	6	7
Temperature (ºC)	15.0	8.0	2.0	-4.5	-10.0	-18.0	-24.0	-31.0

a) Write down Pearson's correlation coefficient correct to 3 decimal places. **1**

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b) Write down the equation of the least squares regression line in the form of $y = mx + b$ where m and b are real numbers correct to 2 decimal places. **1**

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c) Using the results above, describe the correlation between altitude and temperature. **1**

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23. The following table represents a probability distribution.

x	x_1	x_2	x_3	x_4	x_5
$P(X = x)$	0.15	0.15	0.2	k^2	$\frac{k}{2}$

Solve for k. **3**

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24. At the start of the week Susie has three yellow shirts and two green shirts. She wears one clean shirt each day. On Monday, Tuesday and Wednesday of that week, she randomly selects one shirt to wear. In this three day period, what is the probability that Susie does not wear a shirt of the same colour on any two successive days? **2**

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25. Georgina is training for the Tokyo Olympics. She swims 1.5km on the first day, and on each day after that she swims 200 metres more than the previous day. That is, she swims 1.7km on the second day and 1.9km on the third day and so on.

a) How many days of training will she require to have swum a combined total of over 100km? **2**

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Question 25 continues the next page

b) Georgina wants to have swum over 200 km in total over 20 days. If on the first day she still swims 1.5 km but increases her swim by a distance of x km per day. Find the value of x so that she can reach her goal. **1**

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26. By writing $0.\dot{7}\dot{6}$ as a geometric series, use the limiting sum formula to show that $0.\dot{7}\dot{6} = \frac{76}{99}$. **1**

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27. Solve for k such that $y = e^{kt}$ satisfies the equation $\frac{d^2y}{dt^2} = \frac{dy}{dt} + y$. **2**

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28. Find $\frac{dy}{dx}$ if y equals:

a) $\log_2 x$ **1**

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b) 2^x **1**

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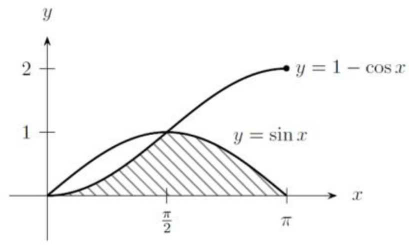
29. The diagram shows the graphs $y = 1 - \cos x$ and $y = \sin x$ for $0 \leq x \leq \pi$. The graphs intersect at $x = \frac{\pi}{2}$.



Find the area of the shaded region. **3**

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30. The table below shows the rate at which water flows into a lake at a specific time.

Time (sec)	0	10	20	30	40	50	60
Rate (litres/sec)	500	400	350	280	200	180	150

By using the trapezoidal rule with 6 subintervals, estimate the total amount of water that flowed into the lake during the first minute. **2**

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31. Consider the curve $y = x(x-2)(x+1)$.

a) Find the coordinates of any stationary point, to 1 decimal place, and determine their nature. **3**

Question 31 continues the next page

b) Find the coordinates of any points of inflection. **3**

Question 31 continues the next page

c) Sketch the curve showing all important features. **2**

d) For what values of c will $x(x-2)(x+1) = c$ have exactly 1 solution? Give your answer correct to 1 decimal place. **1**

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End Section III

Extra Writing Space (Clearly indicate which Question you are doing).

Section IV (30 marks) Student Number:__________________

32. On average, Silvester takes 15 mins to walk to the train station, and from there it takes an average of 45mins to get to the city by train once he is at the station. The standard deviations for the walk and the train trip are 2mins and 5 mins, respectively.

a) Silvester went to meet his friends today. It took him 18mins to walk to the station, and 52mins to get to the city from the station. Calculate the z-scores for the walk and the train trip respectively. **2**

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b) Assume that the walking time from home to the station is normally distributed. Using the empirical rule, find the number of minutes Silvester should allow himself so that he will arrive at the station faster than the allowed time 97.5% of the time? **2**

33. The height of 10 people in centimetres are 170, 180, 185, 188, 192, 193, 193, 194, 196, 202. **2**
Use the interquartile range criteria to determine if there are any outliers in this set of data.

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34. A probability distribution is said to have a **memoryless property** if the probability of some future event occurring is not affected by the occurrence of past events.

In formal statistic terms, a random variable X is said to follow a probability distribution with a memoryless property if for any $a, x \geq 0$ it is true that:

$$P(X > x + a \mid X > a) = P(X > x)$$

You are given that X is modelled by a continuous random variable with probability density function of the form $f(x) = \lambda e^{-\lambda x}$, for some constant λ.

a) Show that the corresponding cumulative distribution function is given by $F(x) = 1 - e^{-\lambda x}$. **2**

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Question 34 continues the next page

b) Hence, prove the memoryless property for the distribution of X: **3**

$$P(X > x + a \mid X > a) = P(X > x)$$

35. Solve $2\log_2 x - 9\log_x 2 = 3$. **3**

36. Sketch the graph $y = \tan\left(\frac{x}{2}\right)$ for $-\pi \leq x \leq \pi$. **2**

37. Consider the series $1 - \tan^2\theta + \tan^4\theta - \tan^6\theta \dots$.

a) For what values of θ in the interval $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ does a limiting sum exist for the series? **2**

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b) Find the expression for this limiting sum in its simplest form. **1**

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38. A barrel maker charges \$50 per hour for the time spent on each job, as well as the cost to cover the power used on machinery. Depending on the setting chosen, the machine uses power at a rate of $20 + \frac{v^2}{10}$ MW per hour, where v barrels/hour is the speed of which the machine makes barrels. (That is, $v = 3$ means the machine can make 3 barrels in an hour). Power costs \$1.25 per MW used. A job is given to make 1200 barrels.

a) Show that the cost (\$$C$) to pay for this job is given by $C = \frac{90000}{v} + 150v$. **2**

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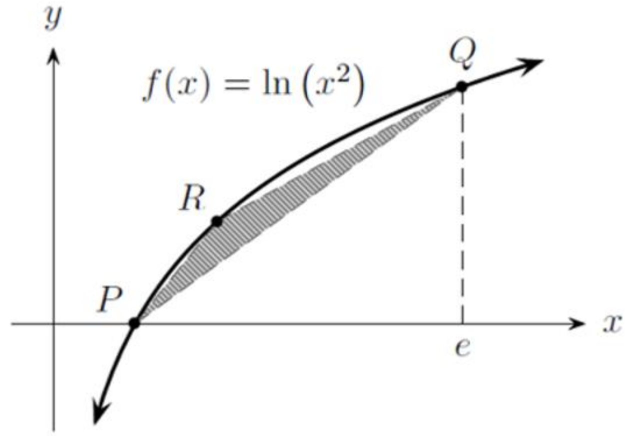
Question continues the next page

b) Assuming that v can take any non-negative values, find the speed v, that minimizes the cost and determine this cost. **3**

39. The diagram shows the graph of the function $y = \ln(x^2)$, where $x > 0$.
The points $P(1, 0)$, $Q(e, 2)$ and $R(t, \ln t^2)$ all lie on the curve.
The area of ΔPQR is maximum when the tangent at R is parallel to the line through P and Q.



a) Show that R has coordinates $(e - 1, \ln[(e - 1)^2])$ for ΔPQR to have maximum area. **2**

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b) Hence find the size of $\angle RPQ$ correct to the nearest degree. **2**

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c) Hence or otherwise, find the maximum area of ΔPQR, correct to 2 decimal places. **2**

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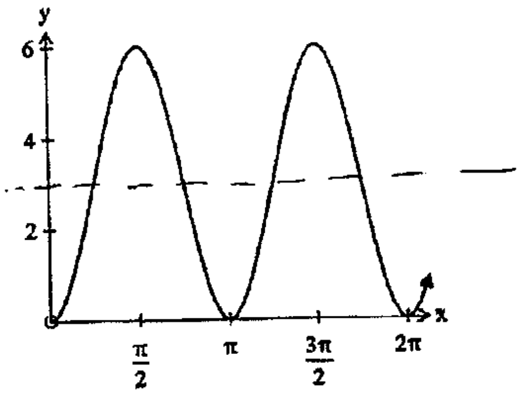
End of Section IV and paper.

Extra Writing Space (Clearly indicate which Question you are doing).

Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1-10.

1. The equation of the graph below is given by $y = A\cos Bx + 3$.



Which of the following are the values of A and B?

A. $A = 3, B = 2$
B. $A = 6, B = 3$
(C.) $A = -3, B = 2$
D. $A = -6, B = \pi$

2. A data set of thirteen scores has a mean and median of 25. The scores 21, 16, 16, 26, 30, 30 are added to this data set.

What of the following MUST BE TRUE?

A. The mean increases
(B.) The mean decreases
C. The median increases
D. The median decreases

• median stays same - same number above & below

scores x Old mean = 13 x 25
= 325

Total new scores = 139
New total = 325 + 139
= 464

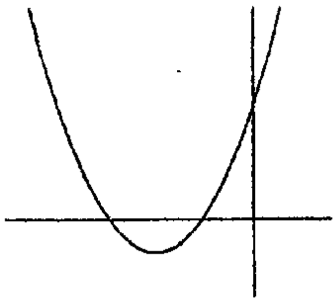
∴ New Mean = 464 ÷ 19
= 24.42

3. Which diagram below best shows the graph $y = 2 - (x + 3)^2$?

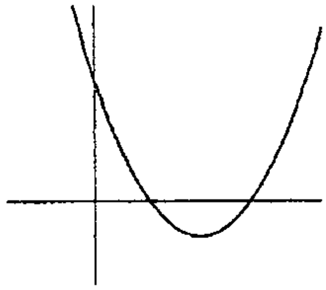
concave down.

$V = (-3, 2)$

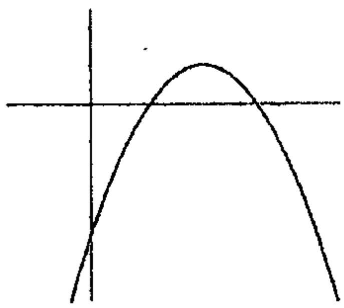
A.



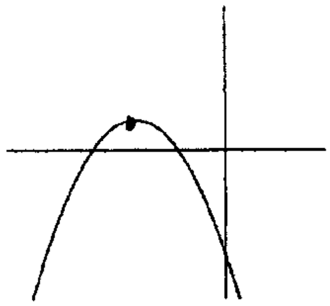
B.



C.



(D.)



4. The following back-to-back stem and leaf plot displays the test results of a class of 24 students.

Boys					Girls				
			1	2	1	2	4		
			3	3	0	2	3	5	
	9	7	4	4	4	5	5	9	9
4	6	2	2	5	3				
				6	1	9			

What is the median and the mode for the class?

(A.) Median is 45 and Mode is 49
B. Median is 46 and Mode is 44
C. Median is 46 and Mode is 49
D. Median is 45 and Mode is 44

5. Which of the following is not equivalent to $\sqrt{(x-1)^2}$?

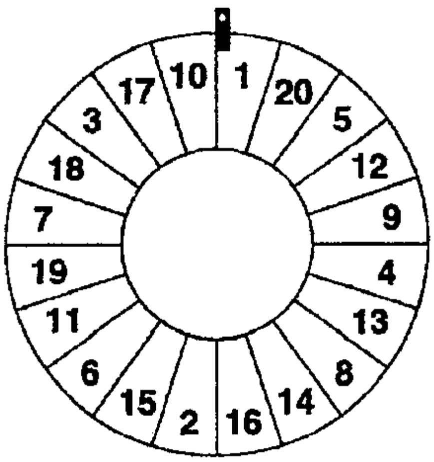
A. $|1-x|$

B. Distance from x to 1 on the number line?

C. $\begin{cases} x-1 & when\, x \geq 1 \\ 1-x & when\, x < 1 \end{cases}$

(D.) $x-1$

6. The wheel below displays numbers 1-20 exactly once for each number.



$\frac{1}{4} \times 160$

$= 40$

If the wheel is spun 160 times, how many times would you expect a number greater than 15 to be obtained?

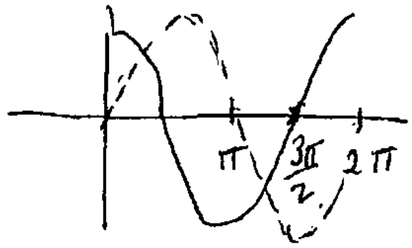
A. 24

B. 32

(C.) 40

D. 48

7. What is $\lim_{h \to 0} \frac{\cos\left(\frac{3\pi}{2}+h\right)-\cos\left(\frac{3\pi}{2}\right)}{h}$? ie $\frac{d}{dx}\cos x = -\sin x$



A. -1

B. 0

(C.) 1

D. Undefined

$f(x) = \cos x$

$f'(x) = -\sin x$

$f'\left(\frac{3\pi}{2}\right) = -\sin \frac{3\pi}{2}$

8. The temperature of a room, in degrees Celsius, is modelled by $T(t)$ where t is the number of minutes after its thermostat is adjusted. What is the best interpretation of $T'(5) = 2$?

(A.) The temperature of the room is increasing at a rate of 2 degrees Celsius per minute, 5 minutes after the thermostat is adjusted.

B. The temperature of the room is increasing at a constant rate of $\frac{2}{5}$ degree Celsius per minute.

C. The temperature of the room increases by 2 degrees Celsius during the first 5 minutes after the thermostat is adjusted.

D. The temperature of the room is 2 degrees, 5 minutes after the thermostat is adjusted.

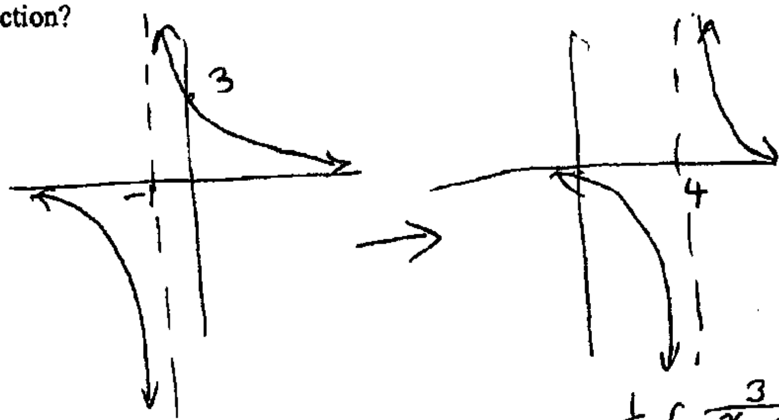
9. The graph of $y = \frac{3}{x+1}$ is translated 4 units right and dilated vertically by a factor of $\frac{1}{2}$. Which of the following gives the equation of the new function?

A. $\frac{y}{2} = \frac{3}{x-3}$

B. $2y = \frac{3}{x-4}$

(C.) $2y = \frac{3}{x-3}$

D. $\frac{y}{2} = \frac{3}{x-4}$

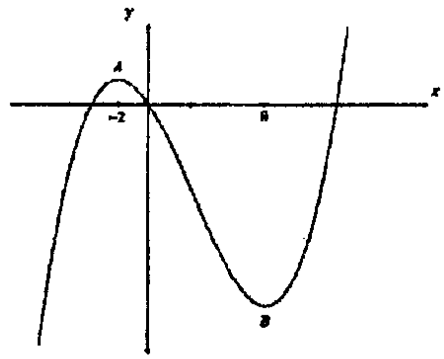


$y = \frac{1}{2}\left(\frac{3}{x-3}\right)$

ie $2y = \frac{3}{x-3}$

ie $\frac{y}{\frac{1}{2}} = \frac{3}{x-3}$

10. The following diagram of $y = f(x)$, has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 8$.



> 0 = 0 concave down ∴ < 0

What is the order of $f(-2)$, $f'(8)$, $f''(-2)$ in ascending order?

A. $f(-2), f'(8), f''(-2)$

B. $f''(-2), f'(8), f(-2)$

C. $f(-2), f''(-2), f'(8)$

D. $f'(8), f''(-2), f(-2)$

End Section I

Section II (30 marks) Student Number:____________________

11. Find the largest domain for which $\sqrt{2x-3}$ is defined. **1**

$2x-3 \geq 0$

$2x \geq 3$

$x \geq \frac{3}{2}$ (1) OR $[\frac{3}{2}, \infty)$

12. Factorise $2x^2 + 3x - 2xy - 3y$. **1**

$x(2x+3) - y(2x+3)$

$(2x+3)(x-y)$ (1)

13. Let $f(x) = -x^2 + x + 4$ and $g(x) = x^2 - 2$.

a) Find $f(3)$. **1**

$f(3) = -(3)^2 + (3) + 4$

$= -9 + 3 + 4$

$= -2$ (1)

b) Express $f(g(x))$ in the form $ax^4 + bx^2 + c$, where a, b and c are non-zero integers. **2**

$g(x) = x^2 - 2$

$f(g(x)) = -(x^2-2)^2 + (x^2-2) + 4$ (1)

$= -(x^4 - 4x^2 + 4) + x^2 - 2 + 4$

$= -x^4 + 4x^2 - 4 + x^2 + 2$

$= -x^4 + 5x^2 - 2$ (1)

14. A circle is given by $x^2 + 2x + y^2 - 4y - 4 = 0$.

a) By completing the square, find the radius and centre of the circle. **2**

$x^2 + 2x + y^2 - 4y - 4 = 0$

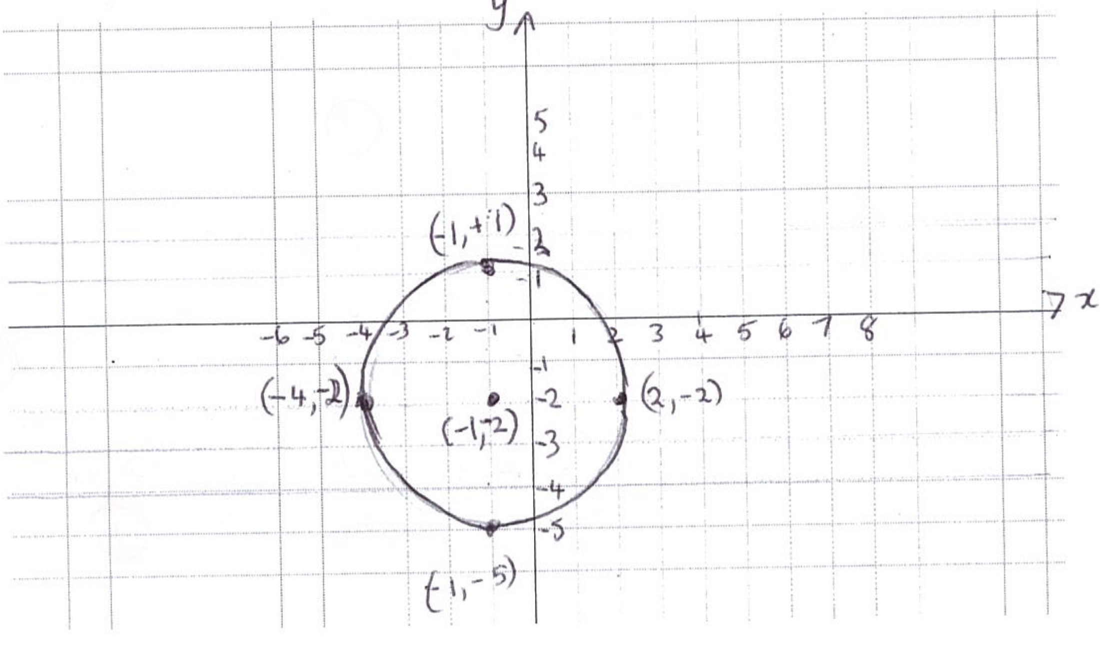
$(x^2 + 2x + 1) + (y^2 - 4y + 4) = 4 + 1 + 4$

$(x+1)^2 + (y-2)^2 = 9$ ①

This is a circle Centre $(-1, 2)$ ①
radius $= 3$

b) This circle is now reflected about the x-axis. Sketch the reflected circle. Do not solve for the coordinate intercepts. **2**

reflected in x axis.
① centre & endpoints
① axis label and scale evident.



15. Rewrite $\sqrt{7}+\sqrt{8}$ in the form $\sqrt{a+b\sqrt{c}}$ using the smallest integer values possible for c. **3**

Let $\sqrt{a+b\sqrt{c}} = \sqrt{7}+\sqrt{8}$

Squaring both sides

$a+b\sqrt{c} = (\sqrt{7}+\sqrt{8})^2$ (1)

$a+b\sqrt{c} = 7 + 2\sqrt{56} + 8$

$a+b\sqrt{c} = 15 + 2\sqrt{56}$

$a+b\sqrt{c} = 15 + 2\sqrt{4\times 14}$ (1)

$a+b\sqrt{c} = 15 + 4\sqrt{14}$

$\therefore \sqrt{a+b\sqrt{c}} = \sqrt{15+4\sqrt{14}}$ (1)

Poorly done. Note: writing $\sqrt{7}+\sqrt{8} = \sqrt{7}+2\sqrt{2}$ not worth anything.

16. An arithmetic sequence is given by 4, 13, 22, 31 … .

a) Find the 25th term of the sequence. **2**

$d = 9$, $a = 4$, $n = 25$ (1)

$T_n = a + (n-1)d$

$T_{25} = 4 + 24\times 9$

$T_{25} = 220$ (1)

b) Find the sum to the 25th term of the sequence. **1**

$l = 220$, $n = 25$, $a = 4$

$S_{25} = \frac{n}{2}(a+l)$

$= \frac{25}{2}(4+220)$

$= 2800$ (1)

17. Given that $4, x, y, \frac{32}{27}$ are four consecutive terms of a geometric series. Solve for x and y. **3**

$\frac{T_2}{T_1} = \frac{T_4}{T_3}$

Method 1: $\frac{x}{4} = \frac{\frac{32}{27}}{y}$

ie $\frac{x}{4} = \frac{32}{27y}$ ---- ①

Also: $\frac{x}{4} = \frac{y}{x}$

ie $x^2 = 4y$

$y = \frac{x^2}{4}$ ---- ②

Substitute $y = \frac{x^2}{4}$ into ① — ①

$\frac{x}{4} = \frac{32}{27\left(\frac{x^2}{4}\right)}$

$\frac{x}{4} = \frac{128}{27x^2}$

$27x^3 = 512$

$x^3 = \frac{512}{27}$ ①

$x = \frac{8}{3}$ When $x = \frac{8}{3}$ $y = \frac{\left(\frac{8}{3}\right)^2}{4}$

$y = \frac{64}{36}$

$\therefore x = \frac{8}{3}$ and $y = \frac{16}{9}$ ①

Method 2:

Using $T_n = ar^n$

$a = 4$

$r = \frac{x}{4}$

$T_4 = \frac{32}{27}$

ie $\frac{32}{27} = 4\left(\frac{x}{4}\right)^3$

$27 \times 4x^3 = 4^3 \times 32$

$x^3 = \frac{32 \times 16}{27}$

$x^3 = \frac{512}{27}$

$x = \frac{8}{3}$ ↗

$y = T_3 = 4 \times \left(\frac{x}{4}\right)^2$

$= \frac{4 \times \left(\frac{8}{3}\right)^2}{16}$

$= \frac{64}{36}$

$y = \frac{16}{9}$

18.

a) Show that the derivative of $\frac{x}{e^x}$ is $\frac{1-x}{e^x}$. **1**

$y = \frac{x}{e^x}$ $\quad u = x \quad v = e^x \quad y' = \frac{vu' - uv'}{v^2}$

$u' = 1 \quad v' = e^x$

Using the quotient rule: $\frac{dy}{dx} = \frac{e^x \cdot 1 - x e^x}{e^{2x}}$

$= \frac{e^x - x e^x}{(e^x)^2}$

$= \frac{e^x(1-x)}{(e^x)^2}$ (1)

$= \frac{1-x}{e^x}$ ← Answer is given so you must clearly show

b) Hence find $\int \frac{1-x+e^x}{2e^x}\,dx$. **2**

$\int \frac{1-x+e^x}{2e^x}\,dx = \int \frac{1-x}{2e^x} + \frac{e^x}{2e^x}\,dx$

$= \frac{1}{2}\int \frac{1-x}{e^x}\,dx + \frac{1}{2}\int \frac{e^x}{e^x}\,dx$

$= \frac{1}{2}\int \left(\frac{1-x}{e^x} + 1\right)dx$ (1)

$= \frac{1}{2}\left[\frac{x}{e^x} + x\right] + c$

$= \frac{x}{2e^x} + \frac{x}{2} + c$ (1)

19. Find the derivative of $x^2 - 1$ using first principles. **3**

$$f'(x) = \lim_{h \to 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{(x+h)^2 - 1 - (x^2 - 1)}{h} \right] \quad (1)$$

$$= \lim_{h \to 0} \left[\frac{x^2 + 2xh + h^2 - 1 - x^2 + 1}{h} \right]$$

$$= \lim_{h \to 0} \left[\frac{2xh + h^2}{h} \right] \quad (1)$$

$$= \lim_{h \to 0} \left[\frac{h(2x+h)}{h} \right]$$

$$= \lim_{h \to 0} (2x+h) \quad (1)$$

$$= 2x$$

Note! you must have the formula correct or 0 marks! This is a critical error so no CFE's!

↖ If you got an answer of $2x$ but did not use 1st principles also 0 marks.

20. The velocity of a particle travelling in a straight line is given by $v = 2t - 4$, where t is time in seconds and x its displacement from the origin in metres. The particle is initially at the origin.

a) Find the expression for the particle's displacement as a function of time. **1**

$v = 2t - 4 \qquad t = 0 \quad x = 0$

$x = \int 2t - 4 \, dt$

$x = \frac{2t^2}{2} - 4t + c$

Subst $t = 0$ $x = 0$ $\therefore c = 0$

$x = t^2 - 4t$

You must evaluate $c = 0$ to get the mark. $x = t^2 - 4t$ is not sufficient.

Question 20 continues the next page

b) Find the total distance travelled for the first 5 seconds. **2**

When $t = 5$ $x = 5^2 - 4 \times 5 = 5$.

Note: 0 marks for an answer of 5.

Stationary point when $v = 0$ ie $2t - 4 = 0$

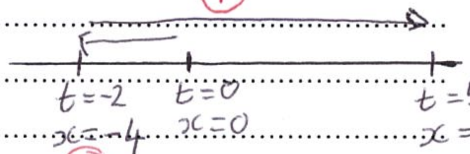
$t = 2$.

∴ particle changes direction when $t = 2$.

When $t = 2$ $x = 2^2 - 4(2)$

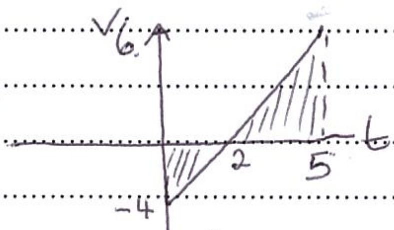
$= -4$.

(1)



ie Particle travels $4 + 9 = 13$ m. (1)

OR



Total distance travelled is area under curve ie $\frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 3 \times 6$

$= 4 + 9$

$= 13$m.

21. Solve for k such that $\int_2^k (-3x + 1)\, dx = \int_k^6 (-3x + 1)\, dx$. **3**

LHS $= \left[\frac{-3x^2}{2} + x\right]_2^k$

$= \frac{-3k^2}{2} + k - \left(\frac{-12}{2} + 2\right)$

(1) - correct integration

$= \frac{-3k^2}{2} + k + 6 - 2$

(1) - correct substitution

$= \frac{-3k^2}{2} + k + 4$

RHS $= \int_k^6 (-3x + 1)\, dx$

$= \left[\frac{-3x^2}{2} + x\right]_k^6$

$= \frac{-108}{2} + 6 - \left(\frac{-3k^2}{2} + k\right)$

$= -48 + \frac{3k^2}{2} - k$.

∴ $\frac{-3k^2}{2} + k + 4 = -48 + \frac{3k^2}{2} - k$

$\frac{6k^2}{2} - 2k - 52 = 0$

$3k^2 - 2k - 52 = 0$

$k = \frac{2 \pm \sqrt{4 - 4 \times 3 \times -52}}{6}$

$k = \frac{2 \pm \sqrt{628}}{6}$ (1)

∴ $k = \frac{1 \pm \sqrt{157}}{3}$

End Section II

Section III (30 marks) Student Number:____________________

22. The data from a weather balloon measuring the air temperature every kilometre as it rises through the atmosphere are shown below:

Altitude (km)	0	1	2	3	4	5	6	7
Temperature (ºC)	15.0	8.0	2.0	-4.5	-10.0	-18.0	-24.0	-31.0

a) Write down Pearson's correlation coefficient correct to 3 decimal places. **1**

$-0.999... = -1.000$

b) Write down the equation of the least squares regression line in the form of $y = mx + b$ where m and b are real numbers correct to 2 decimal places. **1**

$y = -6.52x + 15$

c) Using the results above, describe the correlation between altitude and temperature. **1**

There is a strong negative linear correlation between altitude and temperature

23. The following table represents a probability distribution.

x	x_1	x_2	x_3	x_4	x_5
$P(X = x)$	0.15	0.15	0.2	k^2	$\frac{k}{2}$

Solve for k. **3**

$0.15 + 0.15 + 0.2 + k^2 + \frac{k}{2} = 1$ — (1)

$k^2 + \frac{k}{2} + \frac{1}{2} = 1$

$2k^2 + k + 1 = 2$

$2k^2 + k - 1 = 0$

$(2k-1)(k+1) = 0$

$\therefore k = \frac{1}{2}$ or -1

$k = \frac{1}{2}$ only since $k \geq 0$ — (1)

Don't forget to give a reason when you reject a solution

24. At the start of the week Susie has three yellow shirts and two green shirts. She wears one clean shirt each day. On Monday, Tuesday and Wednesday of that week, she randomly selects one shirt to wear. In this three day period, what is the probability that Susie does not wear a shirt of the same colour on any two successive days? **2**

$P = P(YGY) + P(GYG)$ — (1)

$= \left(\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}\right) + \left(\frac{2}{5} \times \frac{3}{4} \times \frac{1}{3}\right)$

$= \frac{12}{60} + \frac{6}{60}$

$= \frac{18}{60}$

$= \frac{3}{10}$ — (1)

25. Georgina is training for the Tokyo Olympics. She swims 1.5km on the first day, and on each day after that she swims 200 metres more than the previous day. That is, she swims 1.7km on the second day and 1.9km on the third day and so on.

a) How many days of training will she require to have swum a combined total of over 100km? **2**

$a = 1.5 \quad d = 0.2$

$S_n = 100 \Rightarrow \frac{n}{2}\left[2 \times 1.5 + (n-1) \times 0.2\right] = 100$ — (1) Subbing into S_n formula

$n\left(3 + \frac{n-1}{5}\right) = 200$

$3n + \frac{n(n-1)}{5} = 200$

$15n + n(n-1) = 1000$

$n^2 + 14n - 1000 = 0$

$n = \frac{-14 \pm \sqrt{14^2 + 4(1)(-1000)}}{2}$

$= \frac{-14 \pm \sqrt{4196}}{2}$

$n = \frac{-14 + \sqrt{4196}}{2}$ only as $n > 0$ → $n = 25.39$

$\therefore$ She needs 26 days of training — (1) final

Question continues

b) Georgina wants to have swum over 200 km in total over 20 days. If on the first day she still swims 1.5 km but increases her swim by a distance of x km per day. Find the value of x so that she can reach her goal. **1**

$a = 1.5 \quad n = 20, \quad d = x$

$S_{20} = 200 \Rightarrow \frac{20}{2} \times [2 \times 1.5 + (20-1)x] = 200$

$(3 + 19x) = 20$

$19x = 17$

$x = \frac{17}{19}$ — (1)

x must be in km

26. By writing $0.\dot{7}\dot{6}$ as a geometric series, use the limiting sum formula to show that $0.\dot{7}\dot{6} = \frac{76}{99}$. **1**

$0.\dot{7}\dot{6} = 0.76 + 0.0076 + 0.000076 + \ldots$

which is a geometric series with $a = 0.76$, $r = 0.01 < 1$

$\therefore S_\infty = \frac{0.76}{1 - 0.01} = \frac{0.76}{0.99} = \frac{76}{99}$

MUST use a Geometric Series

27. Solve for k such that $y = e^{kt}$ satisfies the equation $\frac{d^2y}{dt^2} = \frac{dy}{dt} + y$. **2**

$y = e^{kt}, \quad \frac{dy}{dt} = ke^{kt}, \quad \frac{d^2y}{dt^2} = k^2e^{kt}$

$\therefore k^2e^{kt} = ke^{kt} + e^{kt}$ — (1)

$k^2 = k + 1$

$k^2 - k - 1 = 0$

$\therefore k = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$ — (1)

28. Find $\frac{dy}{dx}$ if y equals:

a) $\log_2 x$ **1**

$y = \log_2 x$

$= \frac{\ln x}{\ln 2}$

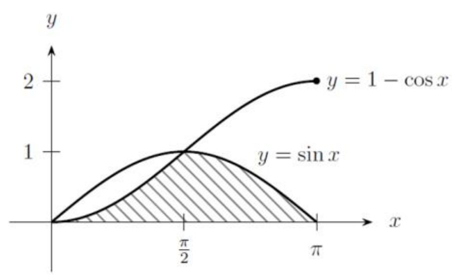
$\therefore \frac{dy}{dx} = \frac{1}{x \ln 2}$

b) 2^x **1**

$y = 2^x$

$\frac{dy}{dx} = (\ln 2) \times 2^x$

29. The diagram shows the graphs $y = 1 - \cos x$ and $y = \sin x$ for $0 \leq x \leq \pi$. The graphs intersect at $x = \frac{\pi}{2}$.



Find the area of the shaded region. **3**

$A = \int_0^{\frac{\pi}{2}} 1 - \cos x \, dx + \int_{\frac{\pi}{2}}^{\pi} \sin x \, dx$ — ① Correct set up

$= \left[x - \sin x\right]_0^{\frac{\pi}{2}} - \left[\cos x\right]_{\frac{\pi}{2}}^{\pi}$ — ① Correctly integrating

$= \left(\frac{\pi}{2} - 1\right) - (0 - 0) - (-1 - 0)$

$= \frac{\pi}{2} u^2$ — ① Final answer

30. The table below shows the rate at which water flows into a lake at a specific time.

Time (sec)	0	10	20	30	40	50	60
Rate (litres/sec)	500	400	350	280	200	180	150

By using the trapezoidal rule with 6 subintervals, estimate the total amount of water that flowed into the lake during the first minute. **2**

Water flowed $\approx \frac{10}{2}[500 + 2(400 + 350 + 280 + 200 + 180) + 150]$ — ①

$= 17350L$ — ①

Section II continues

31. Consider the curve $y = x(x-2)(x+1)$.

a) Find the coordinates of any stationary point, to 1 decimal place, and determine their nature. **3**

$y = x^3 - x^2 - 2x$

$\frac{dy}{dx} = 3x^2 - 2x - 2$

$\frac{dy}{dx} = 0 \Rightarrow 3x^2 - 2x - 2 = 0$ — (1) set $\frac{dy}{dx} = 0$

$\therefore x = \frac{2 \pm \sqrt{4+24}}{6}$, $\therefore x \approx 1.2$ or -0.5

$\frac{d^2y}{dx^2} = 6x - 2$

When $x \approx 1.2$ $y \approx -2.1$ $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} \approx 5.2 > 0$

$\therefore$ local min at $\approx (1.2, -2.1)$ — (1)

when $x \approx -0.5$, $y \approx 0.6$, $\frac{dy}{dx} = 0$, $\frac{d^2y}{dx^2} \approx -5 < 0$

$\therefore$ local max at $(-0.5, 0.6)$ — (1)

MUST have a POINT

b) Find the coordinates of any points of inflection. **3**

$\frac{d^2y}{dx^2} = 0 \Rightarrow 6x - 2 = 0$

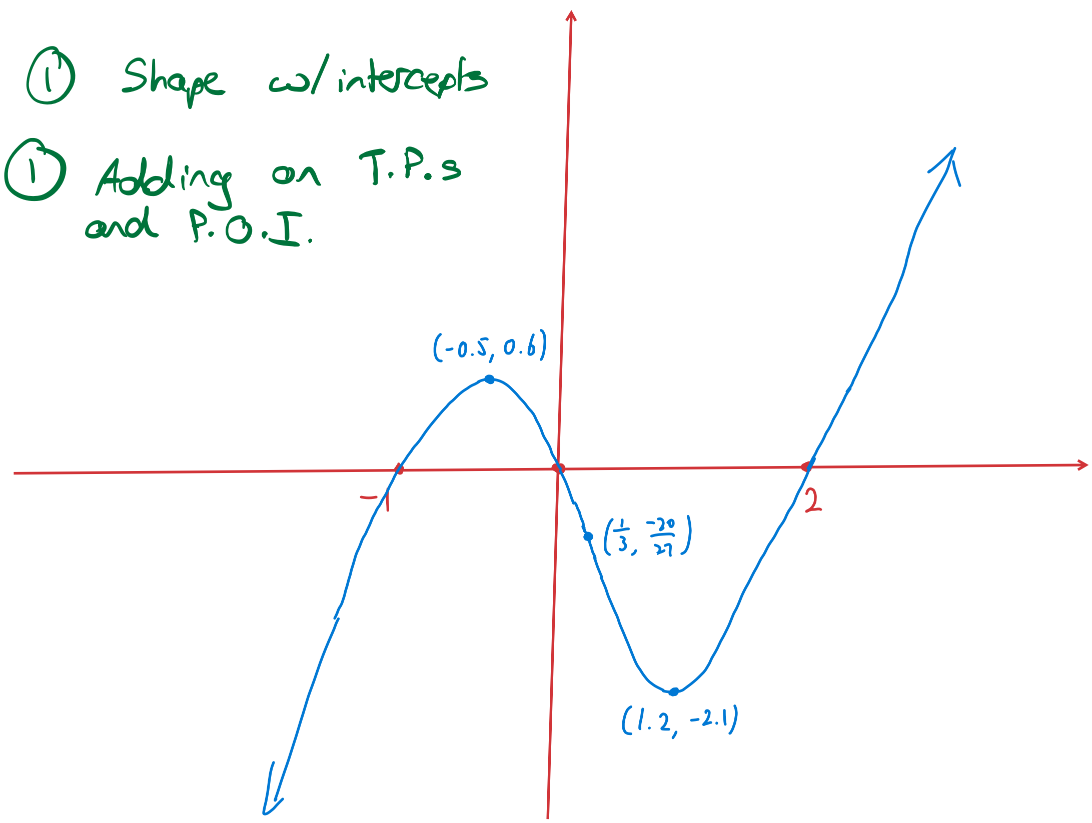
$\therefore x = \frac{1}{3}$ — ① x - value

x	0	$\frac{1}{3}$	1
y''	-2	0	1
	$-$		$+$

When $x = \frac{1}{3}$, $y = \frac{-20}{27}$, $\frac{dy}{dx} \neq 0$, $\frac{d^2y}{dx^2} = 0$ and change of concavity before and after. — ① Checking change of concavity

$\therefore$ point of inflection at $\left(\frac{1}{3}, \frac{-20}{27}\right)$ — ① Final POINT

c) Sketch the curve showing all important features. **2**



d) For what values of c will $x(x-2)(x+1) = c$ have exactly 1 solution? Give your answer correct to 1 decimal place. **1**

$c > 0.6$ or $c < -2.1$

End Section III

Soln Y-12 TRIAL 2022 ADV
Section IV

Section IV (30 marks) Student Number:_________________

32. On average, Silvester takes 15 mins to walk to the train station, and from there it takes an average of 45mins to get to the city by train once he is at the station. The standard deviations for the walk and the train trip are 2mins and 5 mins, respectively.

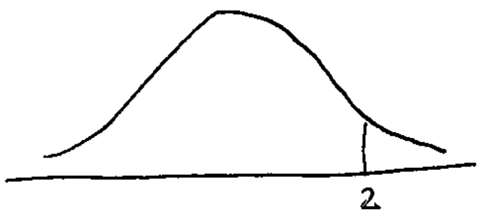
a) Silvester went to meet his friends today. It took him 18mins to walk to the station, and 52mins to get to the city from the station. Calculate the z-scores for the walk and the train trip respectively. **2**

$z_{walk} = \frac{18-15}{2} = 1.5$ # 1m

$z_{train} = \frac{52-45}{2} = 1.4$ # 1m

b) Assume that the walking time from home to the station is normally distributed. Using the empirical rule, find the number of minutes Silvester should allow himself so that he will arrive at the station faster than the allowed time 97.5% of the time? **2**

$97.5\% \Rightarrow z = 2$ 1m



$\frac{x-15}{2} = 2$

$x = 19$ min or

at least 19 min 1m

Student wrote at most 19 min get 1m only

33. The height of 10 people in centimetres are 170, 180, 185, 188, 192, 193, 193, 194, 196, 202. **2**
Use the interquartile range criteria to determine if there are any outliers in this set of data.

IQR = 9 ; $Q_1 = 185$ $Q_3 = 194$

outliers < 185 − 1.5 × 9 i.e. < 171.5
> 194 + 1.5 × 9 i.e. > 207.5 } 1 m

So 170 is the outlier 1 m

34. A probability distribution is said to have a **memoryless property** if the probability of some future event occurring is not affected by the occurrence of past events.

In formal statistic terms, a random variable X is said to follow a probability distribution with a memoryless property if for any $a, x \geq 0$ it is true that:

$$P(X > x + a \mid X > a) = P(X > x)$$

You are given that X is modelled by a continuous random variable with probability density function of the form $f(x) = \lambda e^{-\lambda x}$, for some constant λ.

a) Show that the corresponding cumulative distribution function is given by $F(x) = 1 - e^{-\lambda x}$. **2**

$F(x) = \int_0^x \lambda e^{-\lambda t} = \left[\frac{\lambda e^{-\lambda t}}{-\lambda}\right]_0^x = \left[-e^{-\lambda t}\right]_0^x$ 1 m

must show

$F(x) = e^0 - e^{-\lambda x}$
$= 1 - e^{-\lambda x}$ # 1 m

Question 34 continues the next page

b) Hence, prove the memoryless property for the distribution of X: **3**

$$P(X > x + a \mid X > a) = P(X > x)$$

$P(X > x+a / x > a) = P((x > x+a) \cap (x > a))$

$= \dfrac{P(x > x+a)}{P(x > a)}$ 1m

$= \dfrac{1-(1-e^{-\lambda(x+a)})}{1-(1-e^{-\lambda a})}$ ↖ must show

$= \dfrac{e^{-\lambda x} \cdot e^{-\lambda a}}{e^{-\lambda a}}$

$= e^{-\lambda x}$ 1m

$P(X > x) = 1 - (1 - e^{-\lambda x}) = e^{-\lambda x}$

$\therefore P(X > x+a / x > a) = P(X > x)$

35. Solve $2\log_2 x - 9\log_x 2 = 3$. **3**

$$2\log_2 x - 9\frac{\log_x 2}{\log_2 x} = 3 \qquad 1m$$

$$2\log_2 x - \frac{9}{\log_2 x} - 3 = 0$$

$$2(\log_2 x)^2 - 3\log_2 x - 9 = 0$$

$$(2\log_2 x + 3)(\log_2 x - 3) = 0 \qquad 1m$$

$$\log_2 x = -3/2 \quad \text{or} \quad \log_2 x = 3$$

$$x = 2^{-3/2} \quad \text{or} \quad 2^3 \qquad 1M$$

$$\left(\text{ie } x = \frac{1}{2\sqrt{2}} \text{ or } 8\right)$$

$$0.3535$$

Alternatively $2(\ln x)^2 - 3\ln 2(\ln x) - 9(\ln 2)^2 = 0$

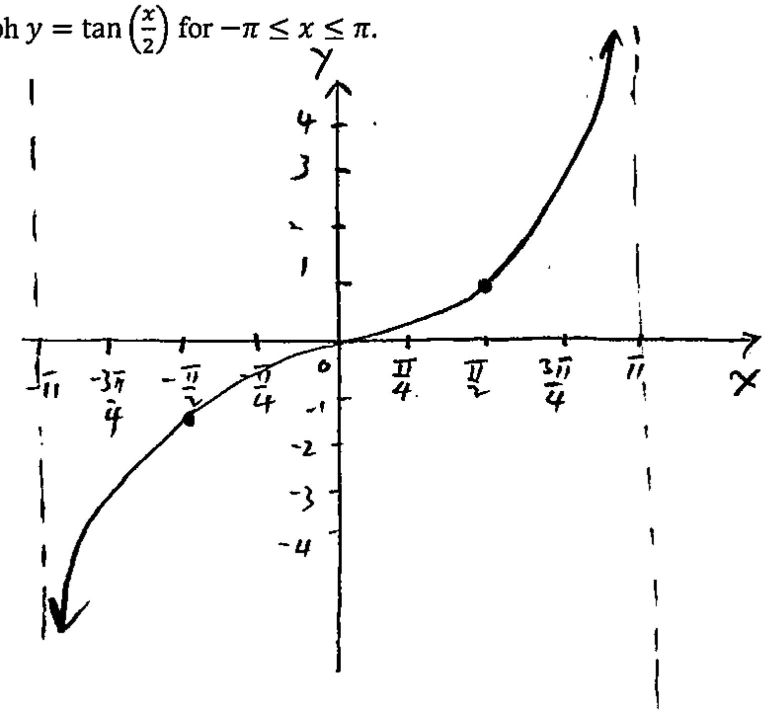
$$\ln x = \frac{3\ln 2 \pm \sqrt{9(\ln 2)^2 - 4(2)(-9(\ln 2)^2)}}{4}$$

$$\ln x = 3\ln 2 \quad \text{or} \quad \frac{-3\ln 2}{2}$$

36. Sketch the graph $y = \tan\left(\frac{x}{2}\right)$ for $-\pi \leq x \leq \pi$. **2**

1 m
Shape with asymptotes $x = \pm\pi$
or
$\left(\frac{\pi}{2}, 1\right), \left(-\frac{\pi}{2}, -1\right)$

2 m
correct graph



37. Consider the series $1 - \tan^2\theta + \tan^4\theta - \tan^6\theta \ldots$.

a) For what values of θ in the interval $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ does a limiting sum exist for the series? **2**

$r = -\tan^2\theta$

$|r| = \tan^2\theta < 1$ 1 m

$-1 < \tan\theta < 1$ ← must show

$-\frac{\pi}{4} < \theta < \frac{\pi}{4}$ 1 m

#

b) Find the expression for this limiting sum in its simplest form. **1**

$S = \frac{1}{1 - -\tan^2\theta}$

$= \frac{1}{1 + \tan^2\theta}$

$= \frac{1}{\sec^2\theta}$

$S = \cos^2\theta$ # 1 m accept $\frac{1}{\sec^2\theta}$

38. A barrel maker charges \$50 per hour for the time spent on each job, as well as the cost to cover the power used on machinery. Depending on the setting chosen, the machine uses power at a rate of $20 + \frac{v^2}{10}$ MW per hour, where v barrels/hour is the speed of which the machine makes barrels. (That is, $v = 3$ means the machine can make 3 barrels in an hour). Power costs \$1.25 per MW used. A job is given to make 1200 barrels.

a) Show that the cost (\$$C$) to pay for this job is given by $C = \frac{90000}{v} + 150v$. **2**

C = cost of wage + cost of machinery

$C = 50T + 1.25 \times \left(20 + \frac{V^2}{10}\right)T$

$T = \frac{1200}{V}$

$\Rightarrow C = 50\left(\frac{1200}{V}\right) + 1.25 \times \left(20 + \frac{V^2}{10}\right)\left(\frac{1200}{V}\right)$ 1M

$C = \frac{60000}{V} + \left(25 + \frac{V^2}{8}\right)\left(\frac{1200}{V}\right)$

$C = \frac{60{,}000}{V} + \frac{30000}{V} + 150V$

$C = \frac{90{,}000}{V} + 150V$ # 1M

Question continues the next page

b) Assuming that v can take any non-negative values, find the speed v, that minimizes the cost and determine this cost. 3

$C' = -\frac{90000}{v^2} + 150 = 0$ 1m

$150 = \frac{90000}{v^2}$

$v^2 = 600$

$v = \sqrt{600}$ $(v > 0)$ 1m $\sqrt{600} \doteqdot 24.5$

$C'' = \frac{2 \times 90000}{v^3} > 0$ as $v > 0$ 1m justification of min

$\therefore$ min C at $v = \sqrt{600}$ × $C = 7348.47$

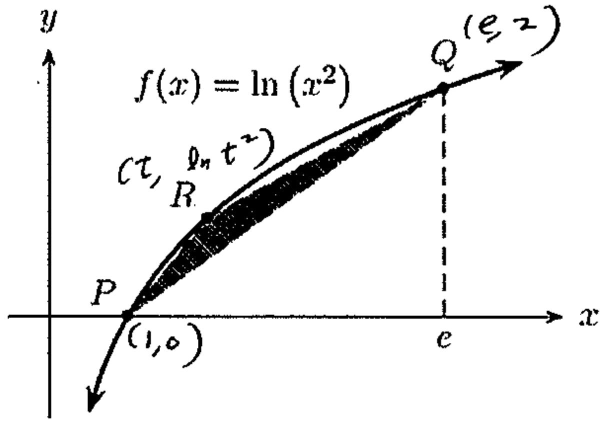
$C = \frac{90000}{\sqrt{600}} + 150 \times \sqrt{600}$

$= \frac{90000}{10\sqrt{6}} + 1500\sqrt{6}$

$= \frac{9000 + 9000}{\sqrt{6}}$

$= \frac{18000}{\sqrt{6}} \doteqdot 7348.47$

39. The diagram shows the graph of the function $y = \ln(x^2)$, where $x > 0$.
The points $P(1,0)$, $Q(e,2)$ and $R(t, \ln t^2)$ all lie on the curve.
The area of ΔPQR is maximum when the tangent at R is parallel to the line through P and Q.



a) Show that R has coordinates $(e-1, \ln[(e-1)^2])$ for ΔPQR to have maximum area. **2**

$f(x) = \ln(x^2)$

$f'(x) = \frac{2x}{x^2} = \frac{2}{x}$ 1m

$m(PQ) = \frac{2-0}{e-1} = \frac{2}{e-1}$

$\therefore \frac{2}{x} = \frac{2}{e-1} \quad \therefore x = e-1$

$y = f(x) = \ln(e-1)^2$

$\Rightarrow R = ((e-1), \ln(e-1)^2)$ 1m

Question continues the next page

b) Hence find the size of $\angle RPQ$ correct to the nearest degree. **2**

Let $m(PQ) = \frac{2}{e-1} = \tan\alpha$ 1m for $m(PQ)$, $m(RP)$

1m for correct angle $7°$ with working

$\alpha = 49°20'$

$m(RP) = \frac{\ln(e-1)^2 - 0}{e-1-1} = \frac{\ln(e-1)^2}{e-2}$

Let $m(RP) = \tan\beta$ $\beta = 56°26'$

$\angle RPQ = 7°6' \doteqdot \underline{\underline{7°}}$ (nearest degree)

See below for alternative ☆

c) Hence or otherwise, find the maximum area of ΔPQR, correct to 2 decimal places. **2**

$\max|PQR| = \frac{1}{2}ab\sin\theta$ 1m

$= \frac{1}{2}|PQ| \cdot |PR| \sin 7°$ ⇐ plus correct value if at least $|PQ|$ or $|PR|$ correct

$|PQ| = \sqrt{(e-1)^2 + 2^2} = \sqrt{e^2 - 2e + 5} = \sqrt{6.9527} = 2.6368$

$|PR| = \sqrt{(e-2)^2 + (\ln(e-1)^2)^2} \doteqdot \sqrt{1.688} = 1.2992$

$\max|PQR| = \frac{1}{2} \times 2.6368 \times 1.299 \sin 7°$

$= 0.2088$

$\doteqdot \underline{\underline{0.21\ u^2}}$ 1m

☆ b) $\cos\theta = \frac{PR^2 + PQ^2 - RQ^2}{2\,PR \cdot PQ} = \frac{1.688 + 6.9527 - 1.8415}{2\sqrt{1.688 \times 6.9527}}$

End of Section IV and paper.